On Torsion-free Vacuum Solutions of the Model of de Sitter Gauge Theory of Gravity (II)

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It is shown that all torsion-free vacuum solutions of the model of dS gauge theory of gravity are the vacuum solutions of Einstein field equations with the same positive cosmological constant. Furthermore, for the gravitational theories with more general quadratic gravitational Lagrangian (F^2+T^2), the torsion-free vacuum solutions are also the vacuum solutions of Einstein field equations.

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The astronomical observations show that our universe is probably an asymptotically de Sitter(dS) one [1, 2]. It raises the interests on dS gauge theories of gravity. There is a model of dS gravity[1], which was first proposed in the 1970's [3, 4]. The dS gravity can be stimulated from dS invariant special relativity [5, 6, 7] and the principle of localization — the full symmetry of the special relativity as well as the laws of dynamics are both localized [8, 9, 10] — and realized in terms of the dS connection on a kind of totally umbilical submanifolds (under the dS-Lorentz gauge) and Yang-Mills type action [3, 8, 9]. It has been shown [9] that the dS gravity may explain the accelerating expansion and supply a natural transit from decelerating expansion to accelerating expansion without the help of the introduction of matter fields in addition to dust. The different de Sitter spacetimes with nonzero torsion in the dS gravity have been presented in [11]. It has also been shown that all vacuum solutions of Einstein field equations with a cosmological constant are the vacuum solutions of the set of field equations without torsion [8, 10]. In particular, Schwarzschild-dS and Kerr-dS metrics are two solutions. The purpose of the present letter is to show that all vacuum, torsion-free solutions in the dS gravity are the vacuum solutions of Einstein field equations with the same positive cosmological constant. Therefore, one may expect that the dS gravity may pass all solar-system-scale observations and experimental tests for general relativity (GR).

The dS gauge theory of gravity is established based on the following consideration. The non-gravitational theory is de Sitter invariant special relativity. The theory of gravity should follow the principle of localization, which says that the *full symmetry* as well as the *laws of dynamics* are both localized, and the gravitational action takes Yang-Mills-type.

A model of dS gauge theory of gravity has been constructed [3, 4, 8, 9, 10] in terms of the de Sitter connection in the dS-Lorentz frame, which reads[2]

$$(\mathcal{B}^{AB}{}_{\mu}) = \begin{pmatrix} B^{ab}{}_{\mu} & R^{-1}e^a_{\mu} \\ -R^{-1}e^b_{\mu} & 0 \end{pmatrix} \in \mathfrak{so}(1,4). \tag{1}$$

where $\mathcal{B}^{AB}{}_{\mu} = \eta^{BC}\mathcal{B}^{A}{}_{C\mu}$. Its curvature is then

$$(\mathcal{F}^{AB}{}_{\mu\nu}) = \begin{pmatrix} F^{ab}{}_{\mu\nu} + R^{-2}e^{ab}{}_{\mu\nu} & R^{-1}T^a{}_{\mu\nu} \\ -R^{-1}T^b{}_{\mu\nu} & 0 \end{pmatrix}, \tag{2}$$

where $e^a{}_{b\mu\nu} = e^a_\mu e_{b\nu} - e^a_\nu e_{b\mu}, e_{a\mu} = \eta_{ab}e^b_\mu$, $F^{ab}{}_{\mu\nu}$ and $T^a{}_{\mu\nu}$ are the curvature and torsion of the Lorentz connection which are defined as:

$$\begin{aligned} \Omega^a &= d\theta^a + \theta^a{}_b \wedge \theta^b = \frac{1}{2}T^a{}_{\mu\nu}dx^\mu \wedge dx^\nu \\ T^a{}_{\mu\nu} &= \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + B^a{}_{c\mu}e^c_\nu - B^a{}_{c\nu}e^c_\mu, \end{aligned} \tag{3}$$

$$\begin{aligned} \Omega^a{}_b &= d\theta^a{}_b + \theta^a{}_c \wedge \theta^c{}_b = \frac{1}{2}F^a{}_{b\mu\nu}dx^\mu \wedge dx^\nu, \\ F^a{}_{b\mu\nu} &= \partial_\mu B^a{}_{b\nu} - \partial_\nu B^a{}_{b\mu} + B^a{}_{c\mu}B^c{}_{b\nu} - B^a{}_{c\nu}B^c{}_{b\mu}, \end{aligned} \tag{4}$$

where $\theta^a = e^a{}_\mu dx^\mu, \theta^a{}_b = B^a{}_{b\mu}dx^\mu$.

The action for the model of de Sitter gauge theory of gravity with sources takes the form of

$$S_{\rm T} = S_{\rm GYM} \ + \ S_{\rm \mathbf{M}}, \tag{5}$$

where

$$\begin{aligned} S_{\rm GYM} &= \frac{\hbar}{4g^2}\int_{\mathcal{M}} d^4xe\mathbf{Tr}_{dS}(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}) \\ &= -\int_{\mathcal{M}} d^4xe\left[\frac{\hbar}{4g^2}F^{ab}{}_{\mu\nu}F_{ab}{}^{\mu\nu} - \chi(F-2\Lambda) \right. \\ &\qquad \left. -\frac{\chi}{2}T^a{}_{\mu\nu}T_a{}^{\mu\nu}\right] \end{aligned} \tag{6}$$

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[1] Hereafter, the model of dS gauge theory of gravity is called the dS gravity for short in this paper.

[2] The same connection with different gravitational dynamics has also been studied (See, e.g. [12, 13, 14, 15, 16, 17, 18, 19, 20])

is the gravitational Yang-Mills action and S_M is the action of sources with minimum coupling. In Eq.(6), $g = \hbar^{1/2}R^{-1}\chi^{-1/2} \sim 10^{-61}$ is the dimensionless gravitational coupling constant, $e = \det(e^a_\mu)$, $\Lambda = 3/R^2$, $\chi = 1/(16\pi G)$, G is the Newtonian gravitational coupling constant, $F = -\frac{1}{2}F^{ab}{}_{\mu\nu}e_{ab}{}^{\mu\nu}$ is the scalar curvature of the Cartan connection. ($c = 1$.)

The field equations can be given via the variational principle with respect to $e^a{}_\mu, B^{ab}{}_\mu$,

$$T_a{}^{\mu\nu}{}_{||\nu} - F^\mu{}_a + \frac{1}{2}Fe^\mu_a - \Lambda e^\mu_a = 8\pi G(T_{\mathrm{M}a}{}^\mu + T_{\mathrm{G}a}{}^\mu), \tag{7}$$

$$F_{ab}{}^{\mu\nu}{}_{||\nu} = R^{-2}(16\pi G S_{\mathrm{M}ab}{}^\mu + S_{\mathrm{G}ab}{}^\mu). \tag{8}$$

Here, $||$ represents the covariant derivative defined by Christoffel symbol $\{^{\mu}_{\nu\kappa}\}$ and Lorentz connection $B^a{}_{b\mu}$, $F_a{}^\mu = -F_{ab}{}^{\mu\nu}e^b_\nu$.

$$T_{\mathrm{M}a}{}^\mu = -\frac{1}{e}\frac{\delta S_{\mathrm{M}}}{\delta e^a_\mu}, \qquad S_{\mathrm{M}ab}{}^\mu = \frac{1}{2\sqrt{-g}}\frac{\delta S_{\mathrm{M}}}{\delta B^{ab}{}_\mu} \tag{9}$$

are the tetrad form of the stress-energy tensor and spin current for matter field, respectively.

$$T_{\mathrm{G}a}{}^\mu = \hbar g^{-2}T_{\mathrm{F}a}{}^\mu + 2\chi T_{\mathrm{T}a}{}^\mu \tag{10}$$

is the tetrad form of the stress-energy tensor of gravitational field, which can be split into the curvature part

$$T_{\mathrm{F}a}{}^\mu = e^\kappa_a \mathrm{Tr}(F^{\mu\lambda}F_{\kappa\lambda}) - \frac{1}{4}e^\mu_a \mathrm{Tr}(F^{\lambda\sigma}F_{\lambda\sigma}) \tag{11}$$

and torsion part

$$T_{\mathrm{T}a}{}^\mu = e^\kappa_a T_b{}^{\mu\lambda}T^b{}_{\kappa\lambda} - \frac{1}{4}e^\mu_a T_b{}^{\lambda\sigma}T^b{}_{\lambda\sigma}. \tag{12}$$

Similarly, the gravitational spin-current

$$S_{\mathrm{G}ab}{}^\mu = S_{\mathrm{F}ab}{}^\mu + 2S_{\mathrm{T}ab}{}^\mu \tag{13}$$

can also be divided into two parts

$$S_{\mathrm{F}ab}{}^\mu = -e_{ab}{}^{\mu\nu}{}_{||\nu} = Y^\mu{}_{\lambda\nu e_{ab}{}^{\lambda\nu}} + Y^\nu{}_{\lambda\nu}e_{ab}{}^{\mu\lambda}, \tag{14}$$

$$S_{\mathrm{T}ab}{}^\mu = T_{[a}{}^{\mu\lambda}e_{b]\lambda}, \tag{15}$$

where

$$Y^\lambda{}_{\mu\nu} = \frac{1}{2}(T^\lambda{}_{\nu\mu} + T_\mu{}^\lambda{}_\nu + T_\nu{}^\lambda{}_\mu) \tag{16}$$

is the contortion.

For vacuum, torsion-free cases, $\mathcal{F}_{ab}{}^{\mu\nu}$ reduces to the Riemann curvature $\mathcal{R}_{ab}{}^{\mu\nu}$ and the gravitational field equations reduce to

$$\mathcal{R}^\mu{}_a - \frac{1}{2}\mathcal{R}e^\mu_a + \Lambda e^\mu_a = -8\pi G T_{\mathrm{R}a}{}^\mu, \tag{17}$$

$$\mathcal{R}_{ab}{}^{\mu\nu}{}_{;\nu} = 0, \tag{18}$$

where $T_{\mathrm{R}a}{}^\mu = e^\nu_a T_{\mathrm{R}}{}^\mu{}_\nu$ is the torsion-free case of $T_{\mathrm{F}a}{}^\mu$ and called as the tetrad form of the stress-energy tensor of Riemann curvature $\mathcal{R}_{ab}{}^{\mu\nu}$, and a semicolon ; is the covariant derivative defined by the Christoffel symbols and Ricci rotation coefficients. Eq. (17) is the Einstein-like equation, while Eq.(18) is the Yang equation in Stephenson-Kilmister-Yang theory of gravity [21].

The trace of Eq.(17) gives

$$\mathcal{R} = 4\Lambda. \tag{19}$$

It can be shown [10, 22] that

$$T_{\mathrm{R}\mu}{}^\nu = 2C_{\lambda\mu}{}^{\kappa\nu}\mathcal{S}^\lambda_\kappa + \frac{\mathcal{R}}{3}\mathcal{S}^\nu_\mu, \tag{20}$$

where $C_{\lambda\mu\kappa\nu}$ is the Weyl tensor,

$$\mathcal{S}_{\mu\lambda} = \mathcal{R}_{\mu\nu} - \frac{1}{4}\mathcal{R}g_{\mu\nu} \tag{21}$$

is the traceless Ricci tensor. Therefore, Eq.(17) after multiplied by e^a_ν can be rewritten as

$$(\chi + \frac{2}{3}\Lambda)\mathcal{S}_{\mu\nu} + C_{\mu\kappa\nu\lambda}\mathcal{S}^{\kappa\lambda} = 0. \tag{22}$$

Since the torsionless curvature tensor is symmetric with respect to its two-pair indices and satisfies the Bianchi identity, Eq.(18) multiplied by $e^a_\kappa e^b_\lambda$ is equivalent to

$$\mathcal{R}^\mu{}_{\kappa;\lambda} = \mathcal{R}^\mu{}_{\lambda;\kappa}.$$

For the constant Ricci scalar, it can be rewritten as

$$\mathcal{S}^\mu{}_{\kappa;\lambda} = \mathcal{S}^\mu{}_{\lambda;\kappa}. \tag{23}$$

Bianchi identity together with the irreducible decomposition of Riemann curvature tensor and Eq.(23) results in

$$C_{\mu\nu[\lambda\sigma;\kappa]} = 0. \tag{24}$$

From Eq.(23), Ricci identity, and the irreducible decomposition of Riemann curvature tensor, one can obtain

$$\begin{aligned} 0 &= (\mathcal{S}^\mu{}_{\kappa;\lambda\rho} - \mathcal{S}^\mu{}_{\lambda;\kappa\rho}) + (\mathcal{S}^\mu{}_{\rho;\kappa\lambda} - \mathcal{S}^\mu{}_{\kappa;\rho\lambda}) \\ &\quad + (\mathcal{S}^\mu{}_{\lambda;\rho\kappa} - \mathcal{S}^\mu{}_{\rho;\lambda\kappa}) \\ &= C^\mu{}_{\sigma\lambda\rho}\mathcal{S}^\sigma{}_\kappa + C^\mu{}_{\sigma\kappa\lambda}\mathcal{S}^\sigma{}_\rho + C^\mu{}_{\sigma\rho\kappa}\mathcal{S}^\sigma{}_\lambda. \end{aligned}$$

This is the integrability condition of Eq.(18)

$$C^*{}_{\mu\lambda\nu\kappa}\mathcal{S}^{\lambda\kappa} = 0, \tag{25}$$

where * is the Hodge star.

Eqs.(22), (24), (25) and the traceless condition of $\mathcal{S}_{\mu\nu}$ constitute a set of equations for vacuum, torsion-free solutions of the dS gravity. The set of the equations are the same as those in Ref.[23] except $\mathcal{R}_{\mu\nu}$'s have been replaced by $\mathcal{S}_{\mu\nu}$'s.

The traceless Ricci tensor $\mathcal{S}_{\mu\nu}$ can be classified into 4 algebraic general types in terms of their eigenvalues [24]. In Segré notation, they are $[1, 1\,1\,1]$, $[Z\,\bar{Z}, 1\,1]$, $[2, 1\,1]$, and $[3, 1]$. In the orthogonal tetrad, the traceless Ricci tensors are given as follows. For [1,1 1 1] type,

$$\mathcal{S}_{ab} = \begin{pmatrix} a & & & \\ & -b & & \\ & & -c & \\ & & & -d \end{pmatrix} \tag{26}$$

with $a+b+c+d=0$. For $[Z\,\bar{Z}$,1 1] type,

$$\mathcal{S}_{ab} = \begin{pmatrix} f & -g & & \\ -g & -f & & \\ & & -c & \\ & & & -d \end{pmatrix} \tag{27}$$

with $2f+c+d=0$. For [2,1 1] type,

$$\mathcal{S}_{ab} = \begin{pmatrix} \pm 1 + a & \pm 1 & & \\ \pm 1 & \pm 1 - a & & \\ & & -c & \\ & & & -d \end{pmatrix} \tag{28}$$

with $2a+c+d=0$. For [3,1] type,

$$\mathcal{S}_{ab} = \begin{pmatrix} a & & 1 & \\ & -a & 1 & \\ 1 & 1 & -a & \\ & & & -d \end{pmatrix} \tag{29}$$

with $3a+d=0$.

In the null tetrad $\boldsymbol{l} = \frac{\sqrt{2}}{2}(\boldsymbol{e}^0 + \boldsymbol{e}^1)$, $\boldsymbol{n} = \frac{\sqrt{2}}{2}(\boldsymbol{e}^0 - \boldsymbol{e}^1)$, $\boldsymbol{m} = \frac{\sqrt{2}}{2}(\boldsymbol{e}^2 + i\boldsymbol{e}^3)$, they are

$$\mathcal{S}_{a'b'} = 2 \begin{pmatrix} \Phi_{00} & \Phi_{11} & & \\ \Phi_{11} & e\Phi_{00} & & \\ & & \Phi_{02} & \Phi_{11} \\ & & \Phi_{11} & \Phi_{02} \end{pmatrix} \tag{30}$$

with $e=+1$, $\Phi_{00} = a-b$, $\Phi_{02} = d-c$, $\Phi_{11} = a+b = -c-d$ for $[1, 1\,1\,1]$ type and with $e=-1$, $\Phi_{00} = 2g$, $\Phi_{02} = d-c$, $\Phi_{11} = 2f = -c-d$ for $[Z\,\bar{Z}, 1\,1]$ type;

$$\mathcal{S}_{a'b'} = 2 \begin{pmatrix} \pm 1 & \Phi_{11} & & \\ \Phi_{11} & 0 & & \\ & & \Phi_{02} & \Phi_{11} \\ & & \Phi_{11} & \Phi_{02} \end{pmatrix} \tag{31}$$

with $\Phi_{11} = 2a = -c-d = 0$ and $\Phi_{02} = d-c$ for $[2, 1\,1]$ type; and

$$\mathcal{S}_{a'b'} = 2 \begin{pmatrix} 0 & \Phi_{11} & \frac{1}{2} & \frac{1}{2} \\ \Phi_{11} 0 & 0 & 0 & \\ \frac{1}{2} & 0 & -2\Phi_{11} & \Phi_{11} \\ \frac{1}{2} & 0 & \Phi_{11} & -2\Phi_{11} \end{pmatrix} \tag{32}$$

with $\Phi_{11} = 2a = -a-d = 0 = (a-d)/2$ for $[3, 1]$ type.

Following the analysis of Debney et al in [23], one can prove

$$\mathcal{S}_{\mu\nu} = 0. \tag{33}$$

This is nothing but the vacuum Einstein field equation with the cosmological constant Λ.

In conclusion, all vacuum, torsion-free solutions in the dS gravity must be the solutions of vacuum Einstein field equation with the same positive cosmological constant. Together with the conclusion obtained in [10], we conclude that the set of vacuum, torsion-free solutions in the dS gravity and the solutions of vacuum Einstein field equations with the same cosmological constant are equivalent. Therefore, the dS gravity is expected to pass the observational tests on the scale of a solar system and explain the indirect evidence of the existence of gravitational wave from the observation data on the binary pulsar PSR1913+16.

Furthermore, the conclusion can be generalized to the Lagrangian

$$\begin{aligned} S_G \;=\; & \int d^4x \{\chi(F - 2\Lambda) + \alpha F_{ab}{}^{\mu\nu} F^{ab}{}_{\mu\nu} + \beta T^a{}_{\mu\nu} T_a{}^{\mu\nu} \\ & + \epsilon e^{ab}{}_{\lambda\sigma} e^{cd}{}_{\mu\nu} F_{ab}{}^{\mu\nu} F_{cd}{}^{\lambda\sigma} + \kappa e^b_\sigma e^\mu_c F_{ab\mu\nu} F^{ac\,\nu\sigma} \\ & + \gamma F_a{}^\mu F^a{}_\mu + \delta e^a_\nu e^b_\mu F_a{}^\mu F_b{}^\nu + \lambda e^\lambda_a e^\sigma_b T^a{}_{\mu\lambda} T^{b\mu}{}_\sigma \\ & + \sigma e^\sigma_a e^\mu_b T^a{}_{\mu\lambda} T^{b\lambda}{}_\sigma \}. \end{aligned}$$

The reasons are as follows. For the torsion-free case, the last two terms have no contribution to the vacuum, torsion-free field equations, and the two terms in the second line contribute the same term in the field equations as $F_{ab}{}^{\mu\nu} F^{ab}{}_{\mu\nu}$ does, thus only alter the unimportant coefficients. The above argument is not valid only when the coupling constants α, ϵ, and κ are suitably arranged so that Eq.(18) does not appear. The first two terms in the third line add the term $(\mathcal{R}^\mu_{[a} e^\nu_{b]})_{;\nu}$ in Yang equation and the stress-energy tensor $\mathcal{R}_{\mu\lambda}\mathcal{R}^{\nu\lambda} - \frac{1}{4}\delta^\nu_\mu \mathcal{R}_{\sigma\lambda}\mathcal{R}^{\sigma\lambda}$ in Einstein-like equation. The latter will not change the constancy of Ricci scalar, and thus the former has still the form of Eq.(23). In other words, Eqs. (24), (25) and the traceless condition of $\mathcal{S}_{\mu\nu}$ are not changed. Only Eq.(22) has slightly different form. A detailed calculation shows that it will not destroy the conclusion.

Obviously, the conclusion is still valid if the integral of the second Chern form of the dS connection over the manifold is added in the action. Finally, the similar discussions can be applied to the AdS case as well and the conclusion is still valid if they can be generalized to the AdS gravity.

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